

Problem 1.B. (Solution):

1.B.1: We can suppose that the work of the atmosphere causes the motion of the water around the bubble and finally the kinetic energy of the water is transformed into the electromagnetic energy of the radiation:

$$p\Delta V = \int E \, dN,$$

where $p\Delta V = (10^5 \text{ Pa}) \cdot \left(\frac{4}{3}\pi\right) (R_0^3 - R^3) \approx (10^5 \text{ Pa}) \cdot \left(\frac{4}{3}\pi R_0^3\right) = 2.68 \cdot 10^{-8} \text{ J}$ and

$$\int E \, dN = \alpha(E_{\max} - E_{\min}) \approx \alpha E_{\max} = (3.3 \cdot 10^6) E_{\max}.$$

We get that $E_{\max} = 8.1 \cdot 10^{-15} \text{ J} = 50 \text{ keV}$ (hard X-ray photons).

1.B.2: Let us denote the radius of the bubble R at the moment when the bubble's wall is moving with the speed of sound c . According to the equation of continuity the speed of the water at a point which is at a distance r from the centre of the bubble ($r > R$) is

$$v(r) = c \cdot \frac{R^2}{r^2}.$$

The kinetic energy of the water is

$$E_{\text{kin}} = \int_R^\infty \frac{1}{2} \rho (4\pi r^2) (v(r))^2 \, dr = 2\pi \rho c^2 R^3,$$

which is equal to the work of the atmosphere: $p\Delta V = p \cdot \frac{4\pi}{3} (R_0^3 - R^3)$. It yields

$$R = \frac{R_0}{\sqrt[3]{1 + \frac{3\rho c^2}{2p}}} \approx \frac{R_0}{\sqrt[3]{\frac{3\rho c^2}{2p}}} = \frac{R_0}{32.3} = 1.24 \, \mu\text{m}.$$